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基于虚土桩模型的三维饱和介质中 浮承桩纵向振动特性分析

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摘 要: 基于 Biot 波动理论提出了一种桩底饱和虚土桩模型, 同时考虑桩周、桩底土体三维波动效应及饱和特性, 建立了三维饱和黏弹性土、虚土桩和实体桩完全耦合振动定解问题。采用势函数求解得出饱和土体位移解, 并利用饱和土-桩界面耦合条件, 求解得出桩顶纵向振动动力阻抗解析解。将所得解退化到已有解析解进行对比验证, 并在此基础上对浮承桩纵向振动特性进行参数化分析, 计算结果表明: 桩底饱和土层厚度越大, 桩顶动刚度和动阻尼曲线振幅及共振频率越小, 且当桩底饱和土层厚度增大到一定程度后, 振幅呈现大、小峰值交替现象; 桩周饱和土体孔隙率仅对桩顶动力阻抗曲线振幅产生明显影响, 而桩底饱和土体孔隙率对桩顶动力阻抗曲线共振幅值和共振频率均影响显著; 随桩周、桩底饱和土体剪切模量的增加, 桩顶动力阻抗曲线共振幅值水平平均明显降低, 且受桩周饱和土体剪切模量影响更为突出。

关 键 词: 三维轴对称; 饱和虚土桩; 桩底土; 纵向振动; 动力阻抗

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Longitudinal vibration characteristics of floating pile in three-dimensional saturated soils based on virtual soil pile model

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Abstract: Based on the Biot's theory of wave propagation, a saturated virtual soil pile model is proposed. Considering the three-dimensional wave effect and saturation characteristics of surrounding soil and beneath the pile toe, a three-dimensional system including saturated viscoelastic soil, virtual soil pile and solid pile is established. The analytical solution for displacement of saturated soil is also derived by potential function method. The vertical dynamic impedance at the pile head is obtained using the pile-soil compatibility conditions. Finally, the obtained solution is reduced to verify its validity with existing solutions. Furthermore, extensive parametric analysis is performed to investigate the effects of saturated soil parameters on the vibration characteristics at the pile head. The computational results show that the amplitude and resonance frequency of dynamic impedance at the pile head in saturated soil decrease with increase of saturated soil thickness beneath the pile toe. When the thickness of saturated soil beneath the pile toe increases to a certain extent, the pattern of staggering peak become evident. The porosity of saturated soil beneath the pile toe has significant effects on both the resonance amplitude and resonance frequency of the dynamic impedance at the pile head, while the porosity of saturated surrounding soil only influence the resonance amplitude. With increasing shear modulus of surrounding soil and soil beneath the pile toe, the amplitude of the dynamic impedance at the pile head is significantly reduced, and it is more affected by the shear modulus of surrounding soil.

Keywords: three dimensional axisymmetry; saturated virtual soil pile; soil beneath the pile toe; longitudinal vibration; dynamic impedance

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1 引言

桩基纵向振动特性研究对于桩基抗振防振设计具有重要的实际意义, 由于该问题的复杂性及工程应用价值, 受到国内外学者的广泛关注^[1-5]。对于如何改进桩-土(桩-桩周土和桩-桩底土)相互作用模型已有相关研究重点。桩-桩周土相互作用模型改进方面: Novak 最早将弹簧-阻尼器模型(Winkler 模型)应用到桩基动力分析中^[6], 并在此基础上发展了桩周土平面应变模型^[7]用来考虑土体径向连续性; 进一步地, Nogami^[8]、胡昌斌^[9]等提出了一种三维连续介质模型, 考虑土体的竖向波动效应; 王奎华等^[10]则对三维连续介质模型进行了完善, 提出了可考虑土体径向位移影响的桩周土真三维连续介质模型。此外, 桩-桩底土相互作用模型改进方面: 对于端承桩将桩底土简化为刚性边界即可保证计算精度, 而对于浮承桩现在多将桩底土简化为 Winkler 模型, 该模型存在参数按经验取值且无法考虑桩底土波动效应等诸多问题^[11], 为减小此类问题的影响, Muki 等^[12]提出一种桩底单相均质弹性半空间土体模型, 但该模型仅适用于桩底土体较厚情况, 无法分析桩底土厚度对桩基的影响。因此, 杨冬英等^[13]基于 Nogami^[14]的一些较早研究, 提出了单相虚土桩模型, 吴文兵等^[15]则基于该模型分析了桩底土层厚度、分层情况及桩底土层参数等诸多因素对桩基纵向振动特性的影响规律。

上述研究工作均基于土体单相假设, 无法考虑土体多相性的影响^[16]。基于此点考虑, 李强^[17]、余俊^[18]等将 Biot 动力波动理论^[19]应用到桩基纵向振动特性研究中, 并应用不同方法(势函数法、微分算子法)对饱和土体动力方程进行解耦, 求解得出端承桩纵向振动相关解析解答, 研究了饱和土参数对桩基纵向振动特性的影响规律。在此基础上, 王桂敏^[20]、李强^[21]等基于 Winkle 模型对饱和土中浮承桩纵向振动特性进行了分析。Militano^[22]、

Senjuntichai^[23]和 Lu^[24]等则将 Muki 模型^[12]拓展到饱和土中, 以考虑桩底饱和土体的波动效应。

综上所述, 在对饱和土中浮承桩纵向振动特性进行研究时, 桩底土体采用 Winkle 模型或弹性半空间模型, 其无法合理考虑桩底饱和土体厚度的影响。基于此, 本文提出一种桩底饱和虚土桩模型, 桩周土采用饱和和三维连续介质模型, 利用桩周土-实体桩、桩底土-饱和虚土桩及实体桩-饱和虚土桩的耦合条件求得桩顶动力阻抗解析解, 进而分析了桩底饱和土层厚度、桩身半径、桩周饱和土体孔隙率、桩底饱和土体孔隙率、桩周饱和土体剪切模量和桩底饱和土体剪切模量对桩顶动力阻抗的影响。

2 定解问题

2.1 力学模型

三维饱和连续介质中基于虚土桩模型的力学简化模型如图 1 所示。桩顶作用谐和激振力为 $\hat{P}e^{i\omega t}$, $\hat{P}$ 为激振力幅值, ω 为激振圆频率, $i = \sqrt{-1}$, t 为时间。基岩上土层总厚度为 H , 桩底土层厚为 H^{sp} , 即饱和虚土桩长度为 H^{sp} , 桩长为 H^p 。桩周土对实体桩侧壁摩阻力为 $f^p(z, t)$, 桩底土对饱和虚土桩侧壁摩阻力为 $f^{sp}(z, t)$, c^s 、 k^s 分别为桩底土上层对桩周土作用的 Voigt 模型的阻尼系数和弹簧系数。本文基本假定参照已有文献^[25]相关描述。

2.2 控制方程

饱和土控制方程:

$$\left. \begin{aligned} G_j^* \left(\nabla^2 - \frac{1}{r^2} \right) u_j^r + \left(\lambda_j^c + G_j^* \right) \frac{\partial e_j}{\partial r} - \alpha_j M_j \frac{\partial \zeta_j}{\partial r} &= \rho_j \frac{\partial^2 u_j^r}{\partial t^2} + \rho_j^f \frac{\partial^2 u_j^f}{\partial t^2} \\ G_j^* \nabla^2 u_j^z + \left(\lambda_j^c + G_j^* \right) \frac{\partial e_j}{\partial r} - \alpha_j M_j \frac{\partial \zeta_j}{\partial r} &= \rho_j \frac{\partial^2 u_j^z}{\partial t^2} + \rho_j^f \frac{\partial^2 u_j^f}{\partial t^2} \\ \alpha_j M_j \frac{\partial e_j}{\partial r} - M_j \frac{\partial \zeta_j}{\partial r} &= \rho_j^f \frac{\partial^2 u_j^f}{\partial t^2} + m_j \frac{\partial^2 \zeta_j}{\partial t^2} + b_j \frac{\partial \zeta_j}{\partial t} \\ \alpha_j M_j \frac{\partial e_j}{\partial z} - M_j \frac{\partial \zeta_j}{\partial z} &= \rho_j^f \frac{\partial^2 u_j^f}{\partial t^2} + m_j \frac{\partial^2 \zeta_j}{\partial t^2} + b_j \frac{\partial \zeta_j}{\partial t} \end{aligned} \right\} \quad (1)$$

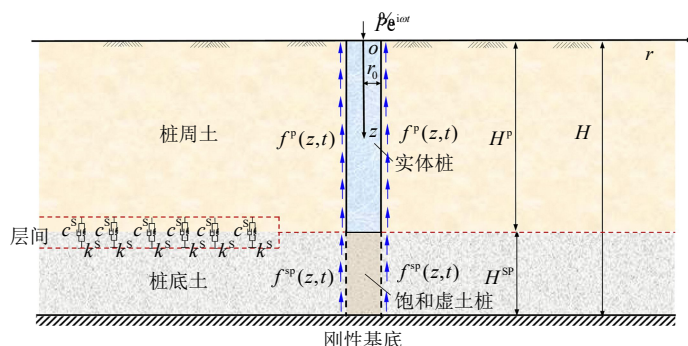


图 1 力学简化模型图

Fig.1 Simplified mechanical model

式中: u_j^r 、 u_j^z 、 w_j^r 、 w_j^z 分别为土骨架径向、纵向位移和流体相对于土骨架径向、纵向位移; $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$; $e_j = \frac{\partial u_j^r}{\partial r} + \frac{u_j^r}{r} + \frac{\partial u_j^z}{\partial z}$; $\zeta_j = -\left(\frac{\partial w_j^r}{\partial r} + \frac{w_j^r}{r} + \frac{\partial w_j^z}{\partial z}\right)$; $\rho_j = (1 - N_j)\rho_j^s + N_j\rho_j^f$ 为饱和土体密度, ρ_j^f 为流体密度, ρ_j^s 为土颗粒密度, N_j 为饱和土孔隙率; $m_j = \rho_j^f/N_j$ 、 $b_j = \xi_j/k_j$ 为黏性耦合系数, ξ_j 为流体黏滞系数, $k_j = \xi_j k_j^D/\rho_j^f g$ 为 Biot 渗透系数, k_j^D 为 Darcy 渗透系数; $G_j^* = G_j(1 + 2\xi_j^* i)$, G_j^* 为饱和土复切变模量, G_j 为剪切模量, ξ_j^* 为阻尼比; $G_j = \lambda_j(1 - 2\nu_j)/2\nu_j$, λ_j 为拉梅常数, ν_j 为泊松比; $\lambda_j^c = \lambda_j + \alpha_j^2 M_j$, α_j 、 M_j 为土颗粒、流体的压缩性常数, $\alpha_j = 1 - K_j^b/K_j^s$, $M_j = (K_j^s)^2/(K_j^d - K_j^b)$, K_j^s 、 K_j^f 、 K_j^b 分别为土颗粒、流体及土骨架的体积压缩模量, $K_j^d = K_j^s[1 + N_j(K_j^s/K_j^f - 1)]$ 。 $j = 1, 2$, $j = 1$ 时对应桩底土参数, $j = 2$ 时对应桩周土参数。

一维饱和土体控制方程 (渗透性较差) [26] 为

$$(\lambda_1 + 2G_1 + \alpha_1^2 M_1) \frac{\partial^2 u^{\text{sp}}}{\partial z^2} = \rho_1 \frac{\partial^2 u^{\text{sp}}}{\partial t^2} \quad (2)$$

式中: u^{sp} 为饱和虚土桩纵向位移; λ_1 、 G_1 、 α_1 、 M_1 和 ρ_1 为桩底土相应参数。

将 $f^{\text{sp}}(z, t)$ 代入式 (2) 可得饱和虚土桩纵向振动控制方程为

$$E^{\text{sp}} A^{\text{sp}} \frac{\partial^2 u^{\text{sp}}}{\partial z^2} - m^{\text{sp}} \frac{\partial^2 u^{\text{sp}}}{\partial t^2} - 2\pi r_0 f^{\text{sp}}(z, t) = 0 \quad (3)$$

式中: $E^{\text{sp}} = \lambda_1 + 2G_1 + \alpha_1^2 M_1$; $A^{\text{sp}} = \pi r_0^2$; $m^{\text{sp}} = A^{\text{sp}} \rho_1$; r_0 为桩身半径。

实体桩纵向振动控制方程为

$$E^{\text{p}} A^{\text{p}} \frac{\partial^2 u^{\text{p}}}{\partial z^2} - m^{\text{p}} \frac{\partial^2 u^{\text{p}}}{\partial t^2} - 2\pi r_0 f^{\text{p}}(z, t) = 0 \quad (4)$$

式中: u^{p} 为实体桩纵向位移; E^{p} 、 r_0 、 A^{p} 和 m^{p} 分别为实体桩弹性模量、半径、截面积和单位长度质量; $A^{\text{p}} = \pi r_0^2$; $m^{\text{p}} = A^{\text{p}} \rho^{\text{p}}$; ρ^{p} 为桩身密度。

2.3 边界条件

(1) 土体

$$u_j^r(\infty, z, t) = 0 \quad (5)$$

桩底土刚性基底竖向位移为 0:

$$u_1^z(r, H, t) = 0 \quad (6)$$

桩周土自由表面正应力为 0:

$$\sigma_2^z(r, 0, t) = \lambda_2 e_2 + 2G_2 \frac{\partial u_2^z}{\partial z} \Big|_{z=0} = 0 \quad (7)$$

式中: $e_2 = \frac{\partial u_2^r}{\partial r} + \frac{u_2^r}{r} + \frac{\partial u_2^z}{\partial z}$ 。

桩底土顶部与桩周土底部为黏弹性支承:

$$\left. \begin{aligned} E_1 \frac{\partial u_1^z}{\partial z} + k^s u_1^z + c^s \frac{\partial u_1^z}{\partial t} \Big|_{z=H^{\text{p}}} &= 0 \\ E_2 \frac{\partial u_2^z}{\partial z} + k^s u_2^z + c^s \frac{\partial u_2^z}{\partial t} \Big|_{z=H^{\text{p}}} &= 0 \end{aligned} \right\} \quad (8)$$

式中: E_1 、 E_2 分别为桩底和桩周土骨架弹性模量,

$$E_1 = \frac{G_1 (3\lambda_1 + 2G_1)}{(\lambda_1 + G_1)}, \quad E_2 = \frac{G_2 (3\lambda_2 + 2G_2)}{(\lambda_2 + G_2)}。$$

(2) 实体桩

$$\frac{\partial u^{\text{p}}}{\partial z} \Big|_{z=0} = -\frac{\beta e^{\text{isot}}}{E^{\text{p}} A^{\text{p}}} \quad (9)$$

(3) 饱和虚土桩

$$u^{\text{sp}} \Big|_{z=H} = 0 \quad (10)$$

(4) 实体桩与饱和虚土桩界面处的边界条件

$$\left. \begin{aligned} u^{\text{p}} \Big|_{z=H^{\text{p}}} &= u^{\text{sp}} \Big|_{z=H^{\text{p}}} \\ E^{\text{p}} A^{\text{p}} \frac{\partial u^{\text{p}}}{\partial z} \Big|_{z=H^{\text{p}}} &= E^{\text{sp}} A^{\text{sp}} \frac{\partial u^{\text{sp}}}{\partial z} \Big|_{z=H^{\text{p}}} \end{aligned} \right\} \quad (11)$$

(5) 桩-土界面条件

桩底土与饱和虚土桩界面:

$$\left. \begin{aligned} u_1^r(r_0, z, t) &= 0 \\ w_1^z(r_0, z, t) &= 0 \end{aligned} \right\} \quad (12)$$

桩周土与实体桩界面不透水, 土骨架径向位移为 0:

$$\left. \begin{aligned} w_2^r(r_0, z, t) &= 0 \\ u_2^r(r_0, z, t) &= 0 \end{aligned} \right\} \quad (13)$$

虚土桩与桩底土、实体桩与桩周土耦合条件为

$$\left. \begin{aligned} u_1^z(r_0, z, t) &= u^{\text{sp}} \quad z \in (H^{\text{p}}, H) \\ \tau_1^{\text{zr}} \Big|_{r=r_0} &= -f^{\text{sp}}(z, t) \\ u_2^z(r_0, z, t) &= u^{\text{p}} \quad z \in (0, H^{\text{p}}) \\ \tau_2^{\text{zr}} \Big|_{r=r_0} &= -f^{\text{p}}(z, t) \end{aligned} \right\} \quad (14)$$

式中: $\tau_1^{zr} = G_1^* \left(\frac{\partial u_1^r}{\partial z} + \frac{\partial u_1^z}{\partial r} \right)$; $\tau_2^{zr} = G_2^* \left(\frac{\partial u_2^r}{\partial z} + \frac{\partial u_2^z}{\partial r} \right)$ 。

3 定解问题求解

3.1 饱和土层振动问题求解

桩顶作用谐和激振力,饱和土层位移满足下式:

$$\left. \begin{aligned} u_j^r(r, z, t) &= \theta_j^r(r, z) e^{i\omega t}, u_j^z(r, z, t) = \theta_j^z(r, z) e^{i\omega t} \\ w_j^r(r, z, t) &= \psi_j^r(r, z) e^{i\omega t}, w_j^z(r, z, t) = \psi_j^z(r, z) e^{i\omega t} \end{aligned} \right\} \quad (15)$$

式中: θ_j^r 、 θ_j^z 、 ψ_j^r 、 ψ_j^z 为饱和土层相应的位移响应幅值。

引入势函数 $\phi_j^s(r, z)$ 、 $\phi_j^f(r, z)$ 、 $\psi_j^s(r, z)$ 、 $\psi_j^f(r, z)$, 并使其满足:

$$\left. \begin{aligned} \theta_j^r &= \frac{\partial \phi_j^s}{\partial r} + \frac{\partial^2 \psi_j^s}{\partial z \partial r}, \quad \theta_j^z = \frac{\partial \phi_j^s}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi_j^s}{\partial r} \right) \\ \psi_j^r &= \frac{\partial \phi_j^f}{\partial r} + \frac{\partial^2 \psi_j^f}{\partial z \partial r}, \quad \psi_j^z = \frac{\partial \phi_j^f}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \psi_j^f}{\partial r} \right) \end{aligned} \right\} \quad (16)$$

将式(15)、(16)代入式(1)并用矩阵形式表示:

$$\left[\begin{array}{cc} (\lambda_j^c + 2G_j^*)\nabla^2 + \rho_j \omega^2 & \alpha_j M_j \nabla^2 + \rho_j^f \omega^2 \\ \alpha_j M_j \nabla^2 + \rho_j^f \omega^2 & M_j \nabla^2 + m_j \omega^2 - b_j i \omega \end{array} \right] \left\{ \begin{array}{c} \phi_j^s \\ \phi_j^f \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\}$$

$$\left[\begin{array}{cc} G_j^* \nabla^2 + \rho_j \omega^2 & \rho_j^f \omega^2 \\ \rho_j^f \omega^2 & m_j \omega^2 - b_j i \omega \end{array} \right] \left\{ \begin{array}{c} \psi_j^s \\ \psi_j^f \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\} \quad (17)$$

式(17)第1式为微分算子方程,则 ϕ_j^s 、 ϕ_j^f 满足方程:

$$\left. \begin{aligned} (\nabla^4 - q_{1j} \nabla^2 + q_{2j}) \phi_j^s &= 0 \\ (\nabla^4 - q_{1j} \nabla^2 + q_{2j}) \phi_j^f &= 0 \end{aligned} \right\} \quad (18)$$

式中:

$$q_{1j} = \frac{2\alpha_j M_j \rho_j^f \omega^2 - (\lambda_j^c + 2G_j^*)(m_j \omega^2 - b_j i \omega) - \rho_j M_j \omega^2}{(\lambda_j^c + 2G_j^*)M_j - \alpha_j^2 M_j^2};$$

$$q_{2j} = \frac{\rho_j \omega^2 (m_j \omega^2 - b_j i \omega) - (\rho_j^f)^2 \omega^4}{(\lambda_j^c + 2G_j^*)M_j - \alpha_j^2 M_j^2}。$$

式(18)可进一步写为

$$\left. \begin{aligned} (\nabla^2 - \beta_{1j}^2)(\nabla^2 - \beta_{2j}^2) \phi_j^s &= 0 \\ (\nabla^2 - \beta_{1j}^2)(\nabla^2 - \beta_{2j}^2) \phi_j^f &= 0 \end{aligned} \right\} \quad (19)$$

$$\text{式中: } \beta_{1j}^2 = \frac{q_{1j} + \sqrt{q_{1j}^2 - 4q_{2j}}}{2}; \beta_{2j}^2 = \frac{q_{1j} - \sqrt{q_{1j}^2 - 4q_{2j}}}{2}。$$

根据算子分解理论对式(19)进行分解可得

$$\left. \begin{aligned} \phi_j^s &= \phi_j^{s1} + \phi_j^{s2} \\ (\nabla^2 - \beta_{1j}^2) \phi_j^{s1} &= 0 \\ (\nabla^2 - \beta_{2j}^2) \phi_j^{s2} &= 0 \end{aligned} \right\} \quad (20)$$

进一步采用分离变量法可得式(19)的解为

$$\left. \begin{aligned} \phi_j^{s1} &= [A_{1j} K_0(h_{1j} r) + B_{1j} I_0(h_{1j} r)] [C_{1j} e^{g_{1j} z} + D_{1j} e^{-g_{1j} z}] \\ \phi_j^{s2} &= [A_{2j} K_0(h_{2j} r) + B_{2j} I_0(h_{2j} r)] [C_{2j} e^{g_{2j} z} + D_{2j} e^{-g_{2j} z}] \end{aligned} \right\} \quad (21)$$

式中: $h_{1j}^2 + g_{1j}^2 = \beta_{1j}^2$; $h_{2j}^2 + g_{2j}^2 = \beta_{2j}^2$; $I_0(hr)$ 、 $K_0(hr)$ 为零阶第1类、第2类虚宗量贝塞尔函数; A_{1j} 、 B_{1j} 、 C_{1j} 、 D_{1j} 、 A_{2j} 、 B_{2j} 、 C_{2j} 、 D_{2j} 为待定常数。

根据边界条件式(5)可知 $B_{1j} = B_{2j} = 0$, 将式(21)代入式(20)第1式可得式(18)第1式的解为

$$\begin{aligned} \phi_j^s &= K_0(h_{1j} r) (C_{1j} e^{g_{1j} z} + D_{1j} e^{-g_{1j} z}) + \\ &K_0(h_{2j} r) (C_{2j} e^{g_{2j} z} + D_{2j} e^{-g_{2j} z}) \end{aligned} \quad (22)$$

同理可得式(18)第2式的解为

$$\begin{aligned} \phi_j^f &= K_0(h_{1j} r) (C_{3j} e^{g_{1j} z} + D_{3j} e^{-g_{1j} z}) + \\ &K_0(h_{2j} r) (C_{4j} e^{g_{2j} z} + D_{4j} e^{-g_{2j} z}) \end{aligned} \quad (23)$$

式中: C_{3j} 、 D_{3j} 、 C_{4j} 、 D_{4j} 为待定常数。

同样地, 采用上述方法对微分算子方程(17)第2式求解可得

$$\left. \begin{aligned} \psi_j^s &= K_0(h_{3j} r) [C_{5j} e^{g_{3j} z} + D_{5j} e^{-g_{3j} z}] \\ \psi_j^f &= K_0(h_{3j} r) [C_{6j} e^{g_{3j} z} + D_{6j} e^{-g_{3j} z}] \end{aligned} \right\} \quad (24)$$

式中: $h_{3j}^2 + g_{3j}^2 = \beta_{3j}^2$;

$$\beta_{3j}^2 = \frac{(\rho_j^f)^2 \omega^4 - \rho_j \omega^2 (m_j \omega^2 - b_j i \omega)}{G_j^* (m_j \omega^2 - b_j i \omega)}; C_{5j}、D_{5j}、C_{6j}、$$

D_{6j} 为待定常数。

根据式(17)的耦合相关性可得:

$$\left. \begin{aligned} C_{3j} &= \gamma_{1j} C_{1j}, \quad C_{4j} = \gamma_{2j} C_{2j}, \quad C_{6j} = \gamma_{3j} C_{5j} \\ D_{3j} &= \gamma_{1j} D_{1j}, \quad D_{4j} = \gamma_{2j} D_{2j}, \quad D_{6j} = \gamma_{3j} D_{5j} \end{aligned} \right\} \quad (25)$$

$$\text{式中: } \gamma_{1j} = \frac{-\alpha_j M_j \beta_{1j}^2 - \rho_j^f \omega^2}{M_j \beta_{1j}^2 + (m_j \omega^2 - b_j i \omega)};$$

$$\gamma_{2j} = \frac{-\alpha_j M_j \beta_{2j}^2 - \rho_j^f \omega^2}{M_j \beta_{2j}^2 + (m_j \omega^2 - b_j i \omega)}; \quad \gamma_{3j} = -\frac{\rho_j^f \omega^2}{m_j \omega^2 - b_j i \omega}。$$

据此可得饱和土层位移为

$$\left. \begin{aligned} \theta_j^0 &= g_{1j} K_0(h_{1j} r) (C_{1j} e^{g_{1j} z} - D_{1j} e^{-g_{1j} z}) + \\ &\quad g_{2j} K_0(h_{2j} r) (C_{2j} e^{g_{2j} z} - D_{2j} e^{-g_{2j} z}) - \\ &\quad h_{3j}^2 K_0(h_{3j} r) (C_{5j} e^{g_{3j} z} + D_{5j} e^{-g_{3j} z}) \\ \theta_j^r &= -h_{1j} K_1(h_{1j} r) (C_{1j} e^{g_{1j} z} + D_{1j} e^{-g_{1j} z}) - \\ &\quad h_{2j} K_1(h_{2j} r) (C_{2j} e^{g_{2j} z} + D_{2j} e^{-g_{2j} z}) - \\ &\quad h_{3j} g_{3j} K_1(h_{3j} r) (C_{5j} e^{g_{3j} z} - D_{5j} e^{-g_{3j} z}) \\ \varphi_j^0 &= \gamma_{1j} g_{1j} K_0(h_{1j} r) (C_{1j} e^{g_{1j} z} - D_{1j} e^{-g_{1j} z}) + \\ &\quad \gamma_{2j} g_{2j} K_0(h_{2j} r) (C_{2j} e^{g_{2j} z} - D_{2j} e^{-g_{2j} z}) - \\ &\quad \gamma_{3j} h_{3j}^2 K_0(h_{3j} r) (C_{5j} e^{g_{3j} z} + D_{5j} e^{-g_{3j} z}) \\ \varphi_j^r &= -\gamma_{1j} h_{1j} K_1(h_{1j} r) (C_{1j} e^{g_{1j} z} + D_{1j} e^{-g_{1j} z}) - \\ &\quad \gamma_{2j} h_{2j} K_1(h_{2j} r) (C_{2j} e^{g_{2j} z} + D_{2j} e^{-g_{2j} z}) - \\ &\quad \gamma_{3j} h_{3j} g_{3j} K_1(h_{3j} r) (C_{5j} e^{g_{3j} z} - D_{5j} e^{-g_{3j} z}) \end{aligned} \right\} \quad (26)$$

进一步地, 对于桩底土将式(26)第 1 式($j=1$)代入式(6)和式(8)第 1 式, 并由 $K_0(h_{11}r)$ 、 $K_0(h_{21}r)$ 、 $K_0(h_{31}r)$ 线性无关性可得

$$g_{1n1} = g_{2n1} = g_{3n1} = g_{n1} \quad n=1, 2, 3 \dots \quad (27)$$

式中: g_{n1} 为满足超越方程 $\frac{e^{2g_{n1}H^p}(g_{n1} + \bar{K}_1^S)}{e^{2g_{n1}H}(g_{n1} - \bar{K}_1^S)} + 1 = 0$ 的无穷多个特征值, $\bar{K}_1^S = (k^S + i\omega c^S)/E_1$ 。

由此可得, D_{11} 、 D_{21} 、 D_{51} 与 C_{11} 、 C_{21} 、 C_{51} 满足如下关系式:

$$D_{11} = e^{2g_{n1}H} C_{11}, \quad D_{21} = e^{2g_{n1}H} C_{21}, \quad D_{51} = -e^{2g_{n1}H} C_{51} \quad (28)$$

将式(26)第 2、3 式分别代入式(12)可得:

$$\left. \begin{aligned} h_{11} C_{11} K_1(h_{11} r_0) + h_{21} C_{21} K_1(h_{21} r_0) + \\ h_{31} g_{n1} C_{51} K_1(h_{31} r_0) = 0 \\ \gamma_{11} g_{n1} C_{11} K_0(h_{11} r_0) + \gamma_{21} g_{n1} C_{21} K_0(h_{21} r_0) - \\ \gamma_{31} h_{31}^2 C_{51} K_0(h_{31} r_0) = 0 \end{aligned} \right\} \quad (29)$$

联立式(29)可将 C_{21} 、 C_{51} 用 C_{11} 表示为

$$\left. \begin{aligned} C_{21} &= -\chi_{21} C_{11} \\ C_{51} &= -\chi_{51} C_{11} \end{aligned} \right\} \quad (30)$$

式中:

$$\begin{aligned} \chi_{21} &= \frac{\gamma_{31} h_{11} h_{31} K_0(h_{31} r_0) K_1(h_{11} r_0) + \gamma_{11} g_{n1}^2 K_0(h_{11} r_0) K_1(h_{31} r_0)}{\gamma_{31} h_{21} h_{31} K_0(h_{31} r_0) K_1(h_{21} r_0) + \gamma_{21} g_{n1}^2 K_0(h_{21} r_0) K_1(h_{31} r_0)}; \\ \chi_{51} &= \frac{\gamma_{21} h_{11} g_{n1} K_0(h_{21} r_0) K_1(h_{11} r_0) - \gamma_{11} h_{21} g_{n1} K_0(h_{11} r_0) K_1(h_{21} r_0)}{\gamma_{21} h_{31} g_{n1}^2 K_0(h_{21} r_0) K_1(h_{31} r_0) + \gamma_{31} h_{31}^2 h_{21} K_0(h_{31} r_0) K_1(h_{21} r_0)} \end{aligned}$$

进一步可得饱和虚土桩与桩底土界面处土骨架纵向振动位移、剪应力幅值为

$$\left. \begin{aligned} \theta_0^0 \Big|_{r=r_0} &= \sum_{n=1}^{\infty} \eta_{n1} C_{n1} (e^{g_{n1} z} - e^{2g_{n1}H} e^{-g_{n1} z}) \\ \theta_0^r \Big|_{r=r_0} &= G_1^* \sum_{n=1}^{\infty} \eta'_{n1} C_{n1} (e^{g_{n1} z} - e^{2g_{n1}H} e^{-g_{n1} z}) \end{aligned} \right\} \quad (31)$$

式中: C_{n1} 为反映桩土耦合作用的一系列待定系数;
 $\eta_{n1} = g_{n1} K_0(h_{1n1} r_0) - g_{n1} \chi_{21} K_0(h_{2n1} r_0) + h_{3n1}^2 \chi_{51} K_0(h_{3n1} r_0)$;
 $\eta'_{n1} = -2h_{1n1} g_{n1} K_1(h_{1n1} r_0) + 2\chi_{21} h_{2n1} g_{n1} K_1(h_{2n1} r_0) +$
 $\chi_{51} h_{3n1} (g_{n1}^2 - h_{3n1}^2) K_1(h_{3n1} r_0)$ 。

对于桩周土则将式(26)第 2 式($j=2$)代入式(8)第 2 式和式(7), 并由 $K_0(h_{12}r)$ 、 $K_0(h_{22}r)$ 、 $K_0(h_{32}r)$ 线性无关性可得

$$g_{1n2} = g_{2n2} = g_{3n2} = g_{n2} \quad n=1, 2, 3 \dots \quad (32)$$

式中: g_{n2} 为满足超越方程 $\frac{g_{n2} \sinh(g_{n2} H^p)}{\bar{K}_2^S \cosh(g_{n2} H^p)} + 1 = 0$ 的

无穷多个特征值, $\bar{K}_2^S = (k^S + i\omega c^S)/E_2$ 。

由此可得, D_{12} 、 D_{22} 、 D_{52} 与 C_{12} 、 C_{22} 、 C_{52} 满足如下关系式:

$$C_{12} + D_{12} = 0, \quad C_{22} + D_{22} = 0, \quad C_{52} - D_{52} = 0 \quad (33)$$

将式(26)第 2、4 式分别代入式(13)可得:

$$\left. \begin{aligned} h_{12} C_{12} K_1(h_{12} r_0) + h_{22} C_{22} K_1(h_{22} r_0) + \\ h_{32} g_{n2} C_{52} K_1(h_{32} r_0) = 0 \\ \gamma_{12} h_{12} C_{12} K_1(h_{12} r_0) + \gamma_{22} h_{22} C_{22} K_1(h_{22} r_0) + \\ \gamma_{32} h_{32} g_{n2} C_{52} K_1(h_{32} r_0) = 0 \end{aligned} \right\} \quad (34)$$

联立式(34)可将 C_{22} 、 C_{52} 用 C_{12} 表示为

$$\left. \begin{aligned} C_{22} &= -\chi_{22} C_{12} \\ C_{52} &= \chi_{52} C_{12} \end{aligned} \right\} \quad (35)$$

其中:

$$\begin{aligned} \chi_{22} &= \frac{(\gamma_{12} - \gamma_{32}) h_{12} K_1(h_{12} r_0)}{(\gamma_{22} - \gamma_{32}) h_{22} K_1(h_{22} r_0)}; \\ \chi_{52} &= \frac{(\gamma_{12} - \gamma_{22}) h_{12} K_1(h_{12} r_0)}{(\gamma_{22} - \gamma_{32}) h_{32} g_{n2} K_1(h_{32} r_0)} \end{aligned}$$

由此可得实体桩与桩周土界面处土骨架纵向振动位移、剪应力幅值为

$$\left. \begin{aligned} \theta_0^0 \Big|_{r=r_0} &= \sum_{n=1}^{\infty} \eta_{n2} C_{n2} \cosh(g_{n2} z) \\ \theta_0^r \Big|_{r=r_0} &= G_2^* \sum_{n=1}^{\infty} \eta'_{n2} C_{n2} \cosh(g_{n2} z) \end{aligned} \right\} \quad (36)$$

式中: C_{n2} 为反映桩土耦合作用的一系列待定系数;

$$\begin{aligned} \eta_{n2} &= 2[g_{n2} K_0(h_{1n2} r_0) - g_{n2} \chi_{22} K_0(h_{2n2} r_0) - \\ &\quad h_{3n2}^2 \chi_{52} K_0(h_{3n2} r_0)]; \end{aligned}$$

$$\eta'_{n2} = -4h_{1n2}g_{n2}K_1(h_{1n2}r_0) + 4\chi_{22}h_{2n2}g_{n2}K_1(h_{2n2}r_0) + 2\chi_{52}h_{3n2}(h_{3n2}^2 - g_{n2}^2)K_1(h_{3n2}r_0)。$$

3.2 桩振动问题求解

综合前述求解所得饱和虚土桩与桩底土界面处土骨架纵向振动位移、剪应力幅值及实体桩与桩周土界面处土骨架纵向振动位移、剪应力幅值,进一步利用桩底土与饱和虚土桩、桩周土与实体桩界面耦合条件,对实体桩桩顶动力阻抗函数进行求解,具体求解流程如图2所示。

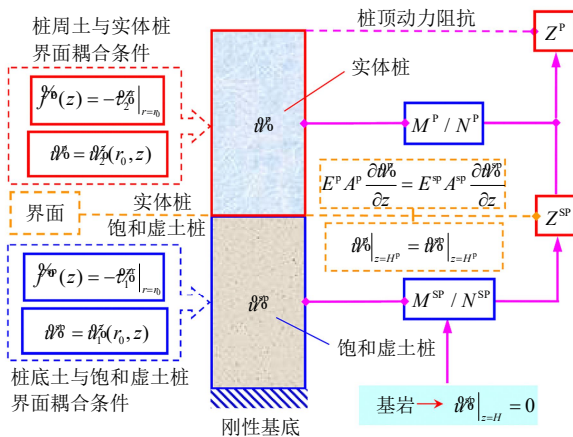


图2 桩顶动力阻抗函数求解流程图

Fig.2 Diagram of solving pile top dynamic impedance

谐和激振作用下饱和虚土桩质点纵向振动位移满足 $u^{\text{sp}}(z, t) = \vartheta^{\text{sp}}(z)e^{i\omega t}$, ϑ^{sp} 为饱和虚土桩质点纵向振动位移响应幅值, 则式(3)可化简为

$$(V^{\text{sp}})^2 \frac{d^2 \vartheta^{\text{sp}}}{dz^2} + \omega^2 \vartheta^{\text{sp}} - \frac{2\pi r_0}{A^{\text{sp}} \rho_1} \dot{\vartheta}^{\text{sp}} = 0 \quad (37)$$

式中: $V^{\text{sp}} = \sqrt{E^{\text{sp}}/\rho_1}$ 。

结合桩底土与饱和虚土桩耦合条件式(14)第2式并将式(31)第2式代入式(37)可得

$$(V^{\text{sp}})^2 \frac{d^2 \vartheta^{\text{sp}}}{dz^2} + \omega^2 \vartheta^{\text{sp}} + \frac{2\pi r_0 G_1^*}{A^{\text{sp}} \rho_1} \sum_{n=1}^{\infty} \eta'_{n1} C_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) = 0 \quad (38)$$

方程(38)对应的齐次方程的通解为

$$\vartheta^{\text{sp}} = M^{\text{sp}} \cos\left(\frac{\omega}{V^{\text{sp}}} z\right) + N^{\text{sp}} \sin\left(\frac{\omega}{V^{\text{sp}}} z\right) \quad (39)$$

$$Z^{\text{sp}} = -E^{\text{sp}} A^{\text{sp}}。$$

$$\frac{M^{\text{sp}}}{N^{\text{sp}}} \left[-\frac{\omega}{V^{\text{sp}}} \sin\left(\frac{\omega}{V^{\text{sp}}} H^{\text{p}}\right) + \sum_{n=1}^{\infty} \kappa_{n1} E_{n1} g_{n1} (e^{g_{n1}H^{\text{p}}} + e^{2g_{n1}H} e^{-g_{n1}H^{\text{p}}}) \right] + \left[\frac{\omega}{V^{\text{sp}}} \cos\left(\frac{\omega}{V^{\text{sp}}} H^{\text{p}}\right) + \sum_{n=1}^{\infty} \kappa_{n1} F_{n1} g_{n1} (e^{g_{n1}H^{\text{p}}} + e^{2g_{n1}H} e^{-g_{n1}H^{\text{p}}}) \right] \\ \frac{M^{\text{sp}}}{N^{\text{sp}}} \left[\cos\left(\frac{\omega}{V^{\text{sp}}} H^{\text{p}}\right) + \sum_{n=1}^{\infty} \kappa_{n1} E_{n1} (e^{g_{n1}H^{\text{p}}} - e^{2g_{n1}H} e^{-g_{n1}H^{\text{p}}}) \right] + \left[\sin\left(\frac{\omega}{V^{\text{sp}}} H^{\text{p}}\right) + \sum_{n=1}^{\infty} \kappa_{n1} F_{n1} (e^{g_{n1}H^{\text{p}}} - e^{2g_{n1}H} e^{-g_{n1}H^{\text{p}}}) \right]$$

(46)

式中: M^{sp} 、 N^{sp} 为待定系数。

方程(38)的特解根据非齐次项形式确定为

$$\vartheta^{\text{sp}*} = \sum_{n=1}^{\infty} \kappa_{n1} C_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) \quad (40)$$

$$\text{式中: } \kappa_{n1} = -\frac{2\pi r_0 G_1^* \eta'_{n1}}{A^{\text{sp}} \rho_1 [(V^{\text{sp}} g_{n1})^2 + \omega^2]}。$$

由此可得方程(38)的解为

$$\vartheta^{\text{sp}} = M^{\text{sp}} \cos\left(\frac{\omega}{V^{\text{sp}}} z\right) + N^{\text{sp}} \sin\left(\frac{\omega}{V^{\text{sp}}} z\right) + \sum_{n=1}^{\infty} \kappa_{n1} C_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) \quad (41)$$

结合饱和虚土桩与桩底土完全耦合条件, 并将式(31)第1式和式(41)代入式(14)第1式, 则有

$$M^{\text{sp}} \cos\left(\frac{\omega}{V^{\text{sp}}} z\right) + N^{\text{sp}} \sin\left(\frac{\omega}{V^{\text{sp}}} z\right) + \sum_{n=1}^{\infty} \kappa_{n1} C_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) = \sum_{n=1}^{\infty} \eta_{n1} C_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) \quad (42)$$

根据固有函数系 $(e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z})$ 在饱和虚土桩桩段 $[H^{\text{p}}, H]$ 内的正交性, 可以确定待定系数 C_{n1} 与 M^{sp} 、 N^{sp} 的关系:

$$C_{n1} = M^{\text{sp}} E_{n1} + N^{\text{sp}} F_{n1} \quad (43)$$

$$\text{式中: } E_{n1} = \frac{L'_{n1}}{(\eta_{n1} - \kappa_{n1}) L_{n1}}; F_{n1} = \frac{L''_{n1}}{(\eta_{n1} - \kappa_{n1}) L_{n1}},$$

$$L_{n1} = \int_{H^{\text{p}}}^H (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z})^2 dz,$$

$$L'_{n1} = \int_{H^{\text{p}}}^H \cos\left(\frac{\omega}{V^{\text{sp}}} z\right) (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) dz,$$

$$L''_{n1} = \int_{H^{\text{p}}}^H \sin\left(\frac{\omega}{V^{\text{sp}}} z\right) (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) dz。$$

则饱和虚土桩纵向振动位移响应幅值可表示为

$$\vartheta^{\text{sp}} = M^{\text{sp}} \left[\cos\left(\frac{\omega}{V^{\text{sp}}} z\right) + \sum_{n=1}^{\infty} \kappa_{n1} E_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) \right] + N^{\text{sp}} \left[\sin\left(\frac{\omega}{V^{\text{sp}}} z\right) + \sum_{n=1}^{\infty} \kappa_{n1} F_{n1} (e^{g_{n1}z} - e^{2g_{n1}H} e^{-g_{n1}z}) \right] \quad (44)$$

将式(44)代入边界条件式(10)可得

$$\frac{M^{\text{sp}}}{N^{\text{sp}}} = -\frac{\sin(H\omega/V^{\text{sp}})}{\cos(H\omega/V^{\text{sp}})} \quad (45)$$

由此可得饱和虚土桩桩顶动力阻抗函数为

同样地, 谐和激振下实体桩质点纵向振动位移 $u^p(z, t) = \vartheta^p(z) e^{i\omega t}$ 。 ϑ^p 为实体桩质点纵向振动位移响应幅值, 且 $f^p(z, t) = \dot{f}^p(z) e^{i\omega t}$, 则式 (4) 可化简为

$$(V^p)^2 \frac{d^2 \vartheta^p}{dz^2} + \omega^2 \vartheta^p - \frac{2\pi r_0}{A^p \rho^p} \dot{f}^p = 0 \quad (47)$$

式中: $V^p = \sqrt{E^p / \rho^p}$ 。

结合桩周土与实体桩耦合条件式 (14) 第 4 式, 并将式 (36) 第 2 式代入式 (47) 可得

$$(V^p)^2 \frac{d^2 \vartheta^p}{dz^2} + \omega^2 \vartheta^p + \frac{2\pi r_0 G_2^*}{A^p \rho^p} \sum_{n=1}^{\infty} \eta'_{n2} C_{n2} \cosh(g_{n2} z) = 0 \quad (48)$$

方程 (48) 对应的齐次方程的通解为

$$\vartheta^p = M^p \cos\left(\frac{\omega}{V^p} z\right) + N^p \sin\left(\frac{\omega}{V^p} z\right) \quad (49)$$

式中: M^p 、 N^p 为待定系数。

方程 (48) 的特解根据非齐次项形式确定为

$$\vartheta^p = \sum_{n=1}^{\infty} \kappa_{n2} C_{n2} \cosh(g_{n2} z) \quad (50)$$

式中: $\kappa_{n2} = -\frac{2\pi r_0 G_2^* \eta'_{n2}}{A^p \rho^p [(V^p g_{n2})^2 + \omega^2]}$ 。

由此可得方程 (48) 的解为

$$\vartheta^p = M^p \cos\left(\frac{\omega}{V^p} z\right) + N^p \sin\left(\frac{\omega}{V^p} z\right) + \sum_{n=1}^{\infty} \kappa_{n2} C_{n2} \cosh(g_{n2} z) \quad (51)$$

结合实体桩与桩周土完全耦合条件, 并将式

(36) 第 1 式和式 (51) 代入式 (14) 第 3 式, 则有:

$$M^p \cos\left(\frac{\omega}{V^p} z\right) + N^p \sin\left(\frac{\omega}{V^p} z\right) +$$

$$\sum_{n=1}^{\infty} \kappa_{n2} C_{n2} \cosh(g_{n2} z) = \sum_{n=1}^{\infty} \eta_{n2} C_{n2} \cosh(g_{n2} z) \quad (52)$$

根据固有函数系 $\cosh(g_{n2} z)$ 在实体桩桩段 $[0, H^p]$ 内的正交性, 可以确定待定系数 C_{n2} 与 M^p 、 N^p 的关系:

$$C_{n2} = M^p E_{n2} + N^p F_{n2} \quad (53)$$

$$\text{式中: } E_{n2} = \frac{L'_{n2}}{(\eta_{n2} - \kappa_{n2}) L_{n2}}; \quad F_{n2} = \frac{L''_{n2}}{(\eta_{n2} - \kappa_{n2}) L_{n2}};$$

$$L_{n2} = \int_0^{H^p} [\cosh(g_{n2} z)]^2 dz;$$

$$L'_{n2} = \int_0^{H^p} \cos\left(\frac{\omega}{V^p} z\right) \cosh(g_{n2} z) dz;$$

$$L''_{n2} = \int_0^{H^p} \sin\left(\frac{\omega}{V^p} z\right) \cosh(g_{n2} z) dz。$$

则实体桩纵向振动位移响应幅值可表示为

$$\begin{aligned} \vartheta^p = & M^p \left[\cos\left(\frac{\omega}{V^p} z\right) + \sum_{n=1}^{\infty} \kappa_{n2} E_{n2} \cosh(g_{n2} z) \right] + \\ & N^p \left[\sin\left(\frac{\omega}{V^p} z\right) + \sum_{n=1}^{\infty} \kappa_{n2} F_{n2} \cosh(g_{n2} z) \right] \end{aligned} \quad (54)$$

由边界条件式 (11) 可得

$$Z^{sp} = -\frac{E^{sp} A^{sp} \frac{\partial \vartheta^p}{\partial z} \Big|_{z=H^p}}{\vartheta^p \Big|_{z=H^p}} = -\frac{E^p A^p \frac{\partial \vartheta^p}{\partial z} \Big|_{z=H^p}}{\vartheta^p \Big|_{z=H^p}} \quad (55)$$

将式 (54) 和式 (46) 代入式 (55) 可得

$$\frac{M^p}{N^p} = \frac{\left[\frac{\omega}{V^p} \cos\left(\frac{\omega}{V^p} H^p\right) + \sum_{n=1}^{\infty} \kappa_{n2} F_{n2} g_{n2} \sinh(g_{n2} H^p) \right] + \frac{Z^{sp}}{E^p A^p} \left[\sin\left(\frac{\omega}{V^p} H^p\right) + \sum_{n=1}^{\infty} \kappa_{n2} F_{n2} \cosh(g_{n2} H^p) \right]}{\left[\frac{\omega}{V^p} \sin\left(\frac{\omega}{V^p} H^p\right) - \sum_{n=1}^{\infty} \kappa_{n2} E_{n2} g_{n2} \sinh(g_{n2} H^p) \right] - \frac{Z^{sp}}{E^p A^p} \left[\cos\left(\frac{\omega}{V^p} H^p\right) + \sum_{n=1}^{\infty} \kappa_{n2} E_{n2} \cosh(g_{n2} H^p) \right]} \quad (56)$$

则实体桩桩顶复阻抗函数为

$$Z^p = -\frac{E^p A^p \omega / V^p}{\frac{M^p}{N^p} (1 + \sum_{n=1}^{\infty} \kappa_{n2} E_{n2}) + \sum_{n=1}^{\infty} \kappa_{n2} F_{n2}} \quad (57)$$

由此得到实体桩桩顶动力阻抗函数为

$$K_d = Z^p = K_r + iK_i \quad (58)$$

式中: K_r 为桩顶动刚度; K_i 为桩顶动阻尼。

4 算例分析

以下算例基于上述推导所得桩顶动力阻抗解析

解答, 各参数按如下取值: 为描述桩底土低渗透性, 取 $k_1^D = 10^{-10} \text{ m/s}$ [26], $k_2^D = 10^{-6} \text{ m/s}$ [27], $H^p = 15 \text{ m}$, $H^{sp} = 2 \text{ m}$, $r_0 = 1.0 \text{ m}$, $\rho^p = 2500 \text{ kg} \cdot \text{m}^3$, $E^p = 25 \text{ GPa}$, $\bar{K}_1^S = 1.0$, 饱和土层参数按表 1 取值。

4.1 本文解退化验证

为验证本文推导所得解析解答的合理性, 将所得解答退化到端承情况 ($H^{sp} \rightarrow 0$) 与李强等 [17] 所得解对比情况如图 3 所示, 退化到单相土情况 ($\rho_j^f \rightarrow 0$ 、 $M_j \rightarrow 0$) [28] 与王奎华等 [29] 所得解对比情况如图 4 所示。由图可见, 本文退化解与李强解 [17]、王奎华解 [29] 吻合。

表 1 饱和土层计算参数
Table 1 Parameters of saturated soils

ρ_j^s	ρ_j^f	k_1^D	k_2^D	N_j	ξ_j	G_j	ξ_j^*	K_j^s	K_j^f	ν_j
/ (kg/m ³)	/ (kg/m ³)	/ (m/s)	/ (m/s)		/ (N · s/m ²)	/ GPa		/ GPa	/ GPa	
2 700	1 000	10 ⁻¹⁰	10 ⁻⁶	0.1	10 ⁻²	0.1	0.05	36	2.0	0.3

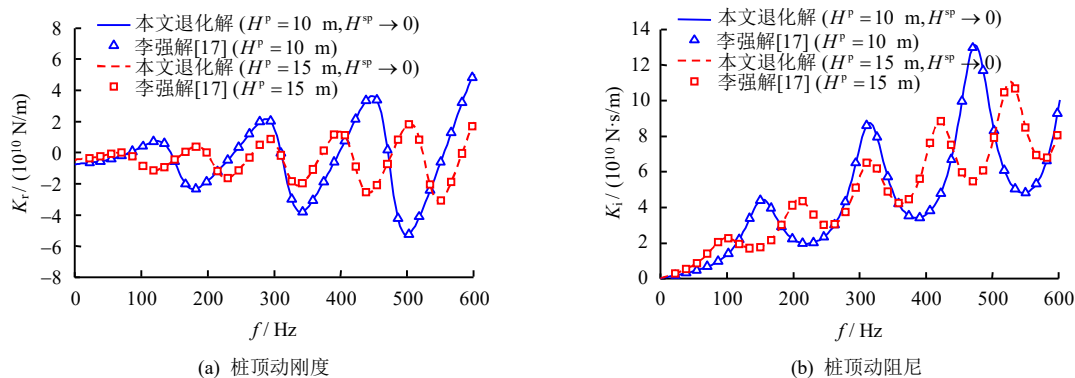


图 3 本文退化解 ($H^{sp} \rightarrow 0$) 与李强^[17]已有解对比
Fig.3 Comparison of present solution ($H^{sp} \rightarrow 0$) with Li's solution

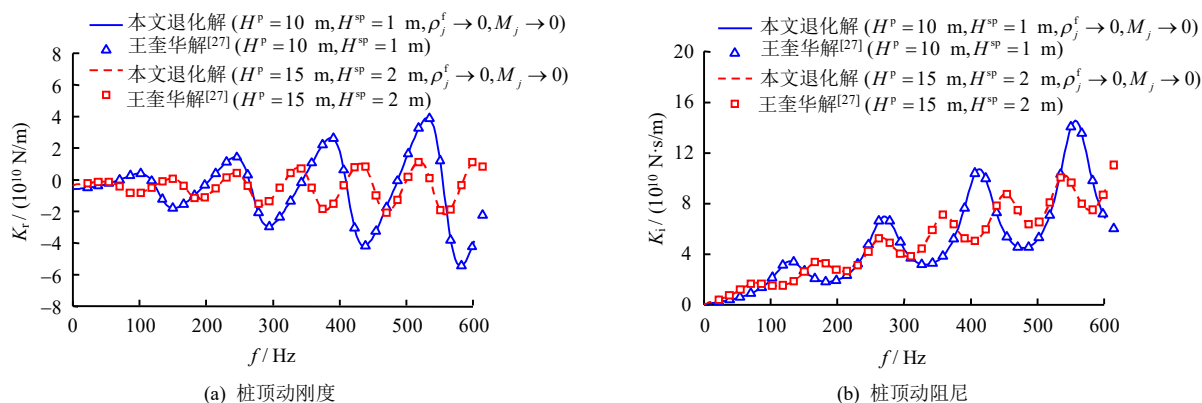


图 4 本文退化解与王奎华^[29]已有解对比
Fig.4 Comparison of present solution with Wang's solution

4.2 桩顶动力阻抗影响因素分析

桩底饱和土层厚度 (即饱和虚土桩长度) 对桩基纵向振动特性的影响如图 5 所示。由图可见, 随着桩底饱和土层厚度的增加, 桩顶动刚度和动阻尼共振幅值及共振频率均明显减小, 且 $H^{sp} = 2$ m 时, 动刚度和动阻尼曲线幅值随频率增加而增大, 而当 $H^{sp} = 4$ m 时, 动刚度和动阻尼曲线则出现幅值大、小交替现象, H^{sp} 越大此种现象越明显。这与文献 [15] 中描述的基于单相虚土桩模型所得桩顶动力阻抗随桩底土层厚度变化规律相差较大, 此种差别存在的原因是桩底饱和虚土桩压缩波速与实体桩更为接近, 与实际饱和土体情况相符。

图 6 所示为桩身半径 (实体桩、饱和虚土桩) 对桩顶动力阻抗曲线的影响规律。由图可见, 随着实体桩及饱和虚土桩的桩身半径增加, 桩顶动力阻

抗曲线的共振幅值水平变高。实体桩及饱和虚土桩的桩身半径变化对桩顶动力阻抗共振频率的影响可以忽略。

桩周和桩底饱和土体孔隙率对桩顶动刚度、动阻尼的影响如图 7 和图 8 所示。由图可见, 桩周饱和土体孔隙率仅对桩顶动刚度、动阻尼曲线的振幅影响较明显, 振幅随桩周饱和土体孔隙率的增加而增大。桩底饱和土体孔隙率不仅影响动刚度、动阻尼曲线的振幅, 对共振频率的影响也不可忽略。另外, 当桩底饱和土体孔隙率增大到 0.4 时, 动刚度、动阻尼曲线呈现典型的大、小峰值交替现象, 且此种现象随桩底饱和土体孔隙率的增加更加明显。

图 9 和图 10 所示分别为桩周、桩底饱和土体剪切模量对桩基纵向振动特性的影响规律。由图可见, 桩周饱和土、桩底饱和土剪切模量对动刚度和动阻

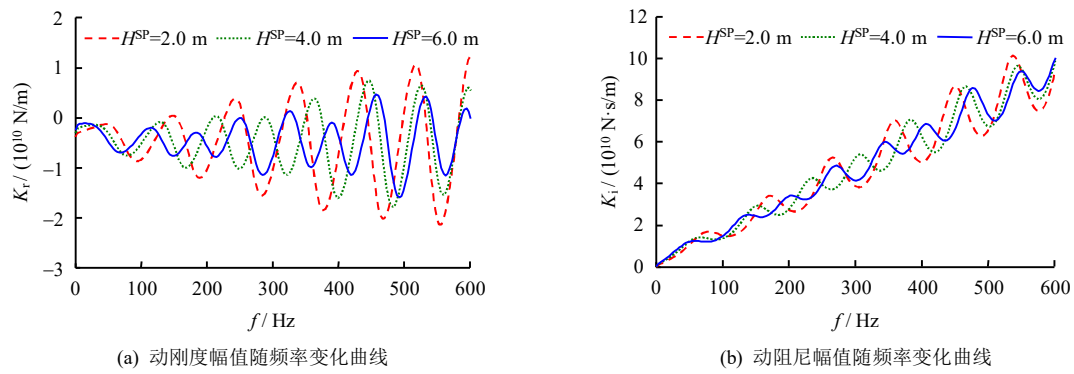


图 5 饱和土层厚度（桩底）对桩基纵向振动特性的影响

Fig.5 Influence of soil thickness (beneath the pile toe) on longitudinal vibration characteristics of pile foundation

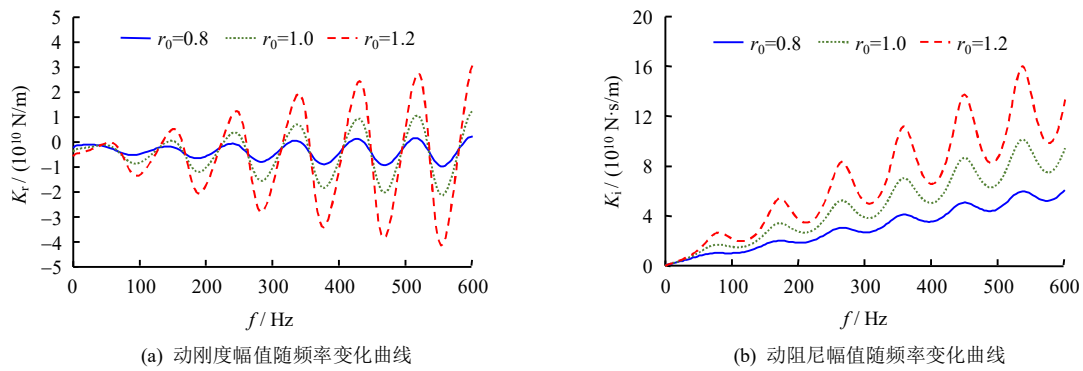


图 6 桩身半径对桩顶动力阻抗影响

Fig.6 Effect of pile radius on dynamic impedance at the pile head

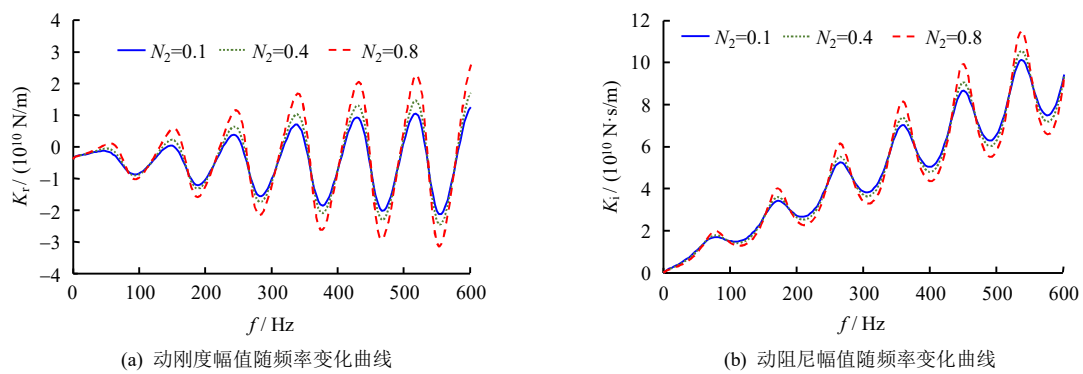


图 7 饱和土体孔隙率（桩周）对桩基纵向振动特性的影响

Fig.7 Effect of porosity (surrounding soil) on longitudinal vibration characteristics of pile foundation

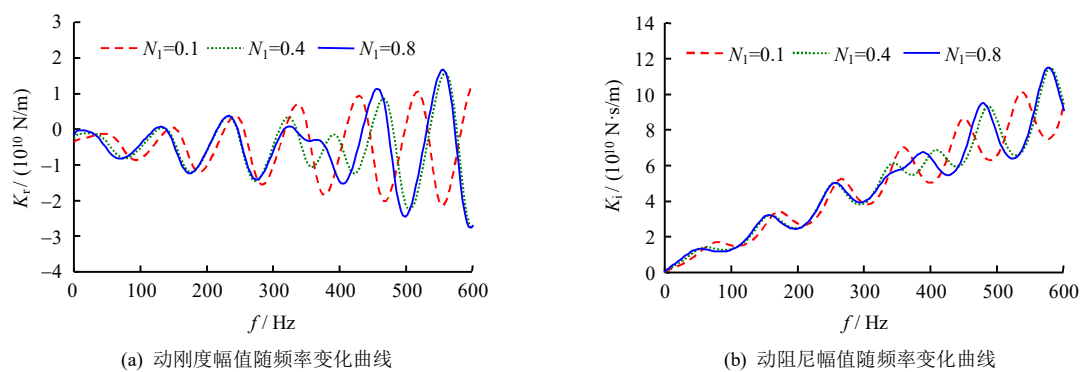


图 8 饱和土体孔隙率（桩底）对桩基纵向振动特性的影响

Fig.8 Effect of porosity (soil beneath the pile toe) on longitudinal vibration characteristics of pile foundation

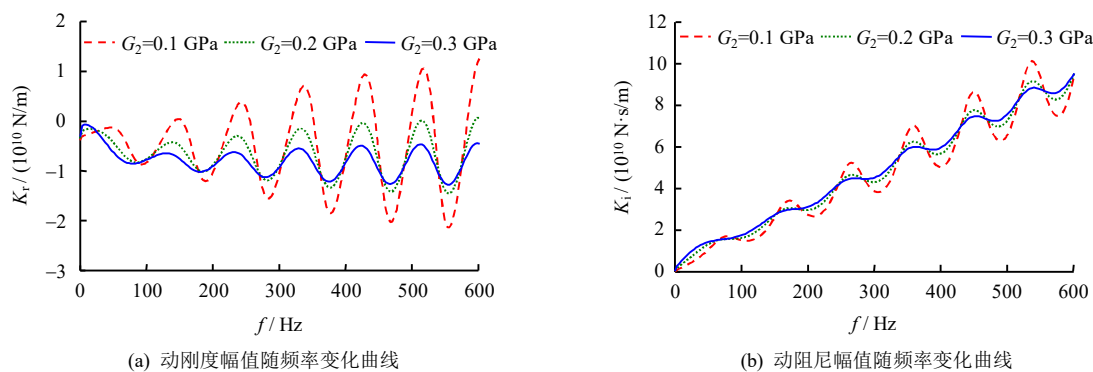


图9 饱和土体剪切模量(桩周)对桩基纵向振动特性的影响

Fig.9 Effect of shear modulus (surrounding soil) on longitudinal vibration characteristics of pile foundation

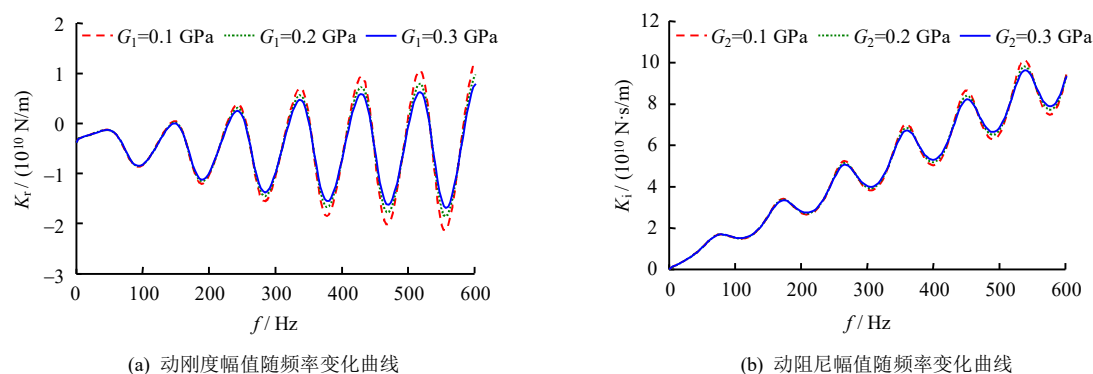


图10 桩底饱和土体剪切模量对桩基纵向振动特性的影响

Fig.10 Effect of shear modulus (soil beneath the pile toe) on longitudinal vibration characteristics of pile foundation

尼曲线共振幅值影响显著。且相对而言, 桩周饱和土体剪切模量变化对桩顶阻抗曲线共振幅值影响更为突出。桩顶动刚度和动阻尼曲线振幅随桩周、桩底饱和土体剪切模量的增加而减小, 表明软土地中桩基较硬土地中桩基抗振效果差。土体剪切模量对动刚度和动阻尼曲线的影响则可忽略。

5 结 论

(1) 桩顶动刚度和动阻尼共振幅值及共振频率均随桩底饱和土层厚度的增加而减小, 且当 $H^{\text{SP}} = 4 \text{ m}$ 时, 动刚度和动阻尼曲线则出现幅值大、小交替现象, H^{SP} 越大此种现象越明显。

(2) 桩顶动力阻抗曲线振幅随着桩身半径(实体桩、饱和虚土桩)增加而增大, 但实体桩及饱和虚土桩的桩身半径变化对桩顶动力阻抗共振频率的影响可以忽略。

(3) 桩周饱和土体孔隙率仅对桩顶动刚度、动阻尼曲线的振幅影响较明显, 而桩底饱和土体孔隙率不仅影响动刚度、动阻尼曲线的振幅, 对共振频率的影响也不可忽略。另外, 当桩底饱和土体孔隙率增大到 0.4 时, 动刚度、动阻尼曲线呈现典型的大、小峰值交替现象, 且此种现象随桩底饱和土体

孔隙率的增加更加明显。

(4) 随桩周、桩底饱和土体剪切模量的增加, 桩顶动力阻抗曲线振幅均明显减小, 且其受桩周饱和土体剪切模量影响更为突出。而土体剪切模量对动刚度和动阻尼曲线的影响则可忽略。

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